Surds

The plan...

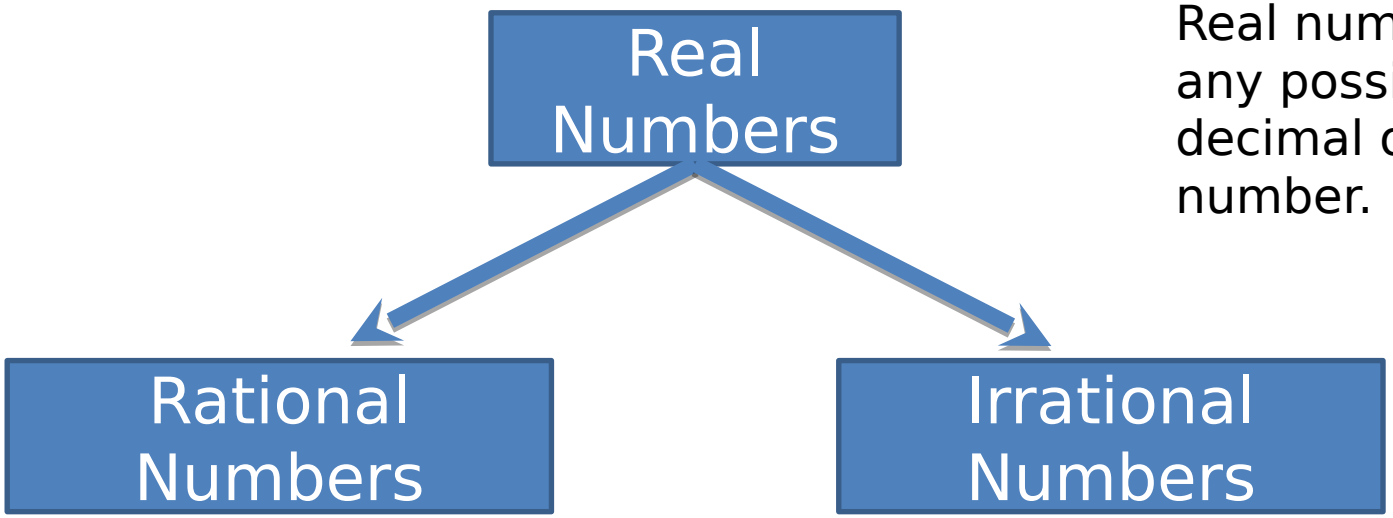
P1.4 Calculation of squares and square roots

C1.4 Calculation of powers and roots

B1.10 Surds (radicals), simplification of square root expressions

A1.10 Rationalisation of the denominator

Types of numbers

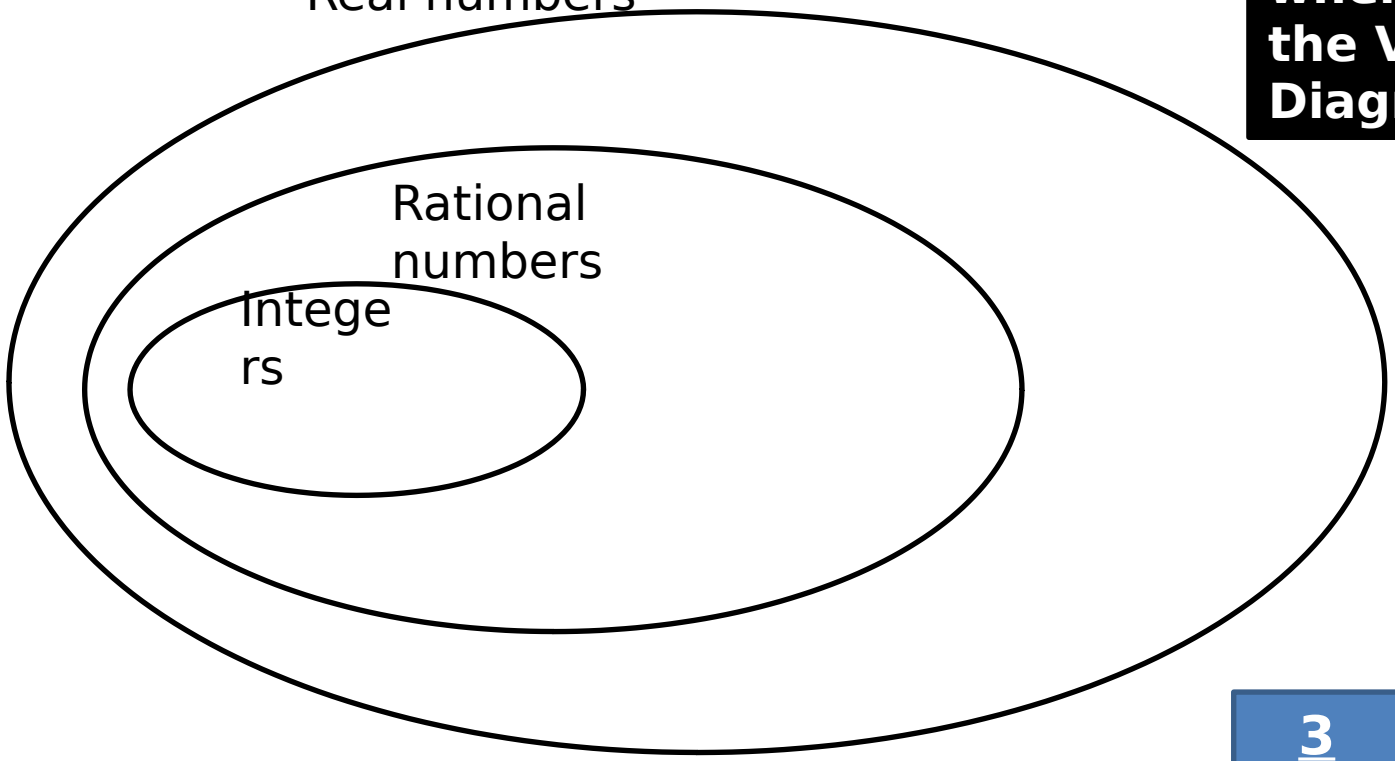


Real numbers are any possible decimal or whole number.

are all numbers which can be expressed as some fraction involving integers (whole numbers), e.g. $\frac{1}{4}$, $3\frac{1}{2}$, -7.

are real numbers which are not rational.

Types of numbers



Click each number to see where it goes in the Venn Diagram.

3

$0.\dot{7}$

π

$1.\dot{3}$

$\sqrt{2}$

-1

$\frac{3}{4}$

$\sqrt{9}$

e

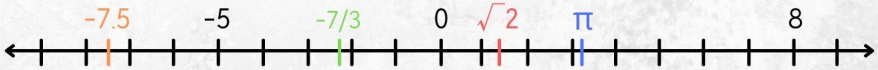
(Click the blue boxes above)

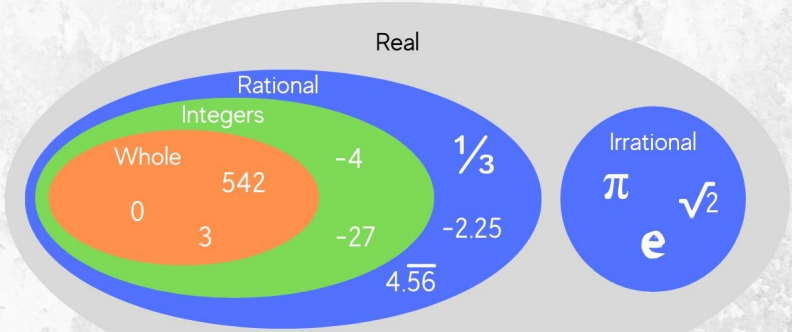
Integers Examples

- Negative : -1, -2, -3, -4, -5 ...
- Non-negative : 0, 6, 7, 8, 9 ...
- Positive : 1, 2, 3, 4, 5 ...
- Zero : 0 all by itself

Real Numbers

A real number is any number that can be represented on a number line or by an infinite decimal expansion.





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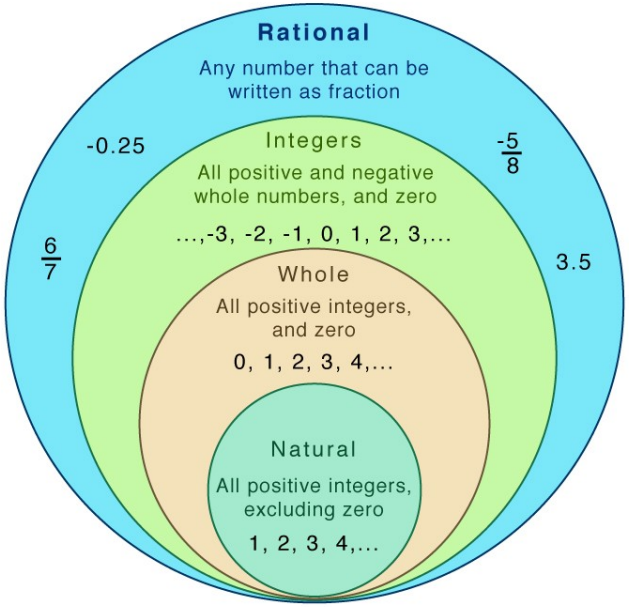
Rational Numbers

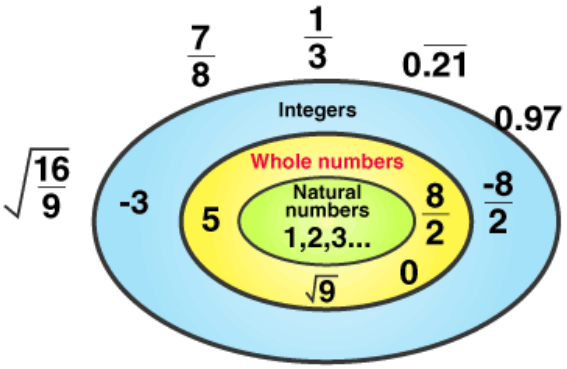
MATH MONKS



RATIONAL VS. IRRATIONAL NUMBERS

BYJU'S The Learning App

RATIONAL NUMBERS



IRRATIONAL NUMBERS

$\sqrt{8}$

$\sqrt{1.6}$ $\sqrt{11}$

$\sqrt{\frac{2}{5}}$ π

0.3030030003

Real but irrational numbers: cannot be represented as a simple

Non-Real Numbers

Imaginary Numbers:

If there is a negative inside the radical, then it is a

NON-REAL NUMBER

$\sqrt{-25} = 5i$

$\sqrt{-9} = 3i$

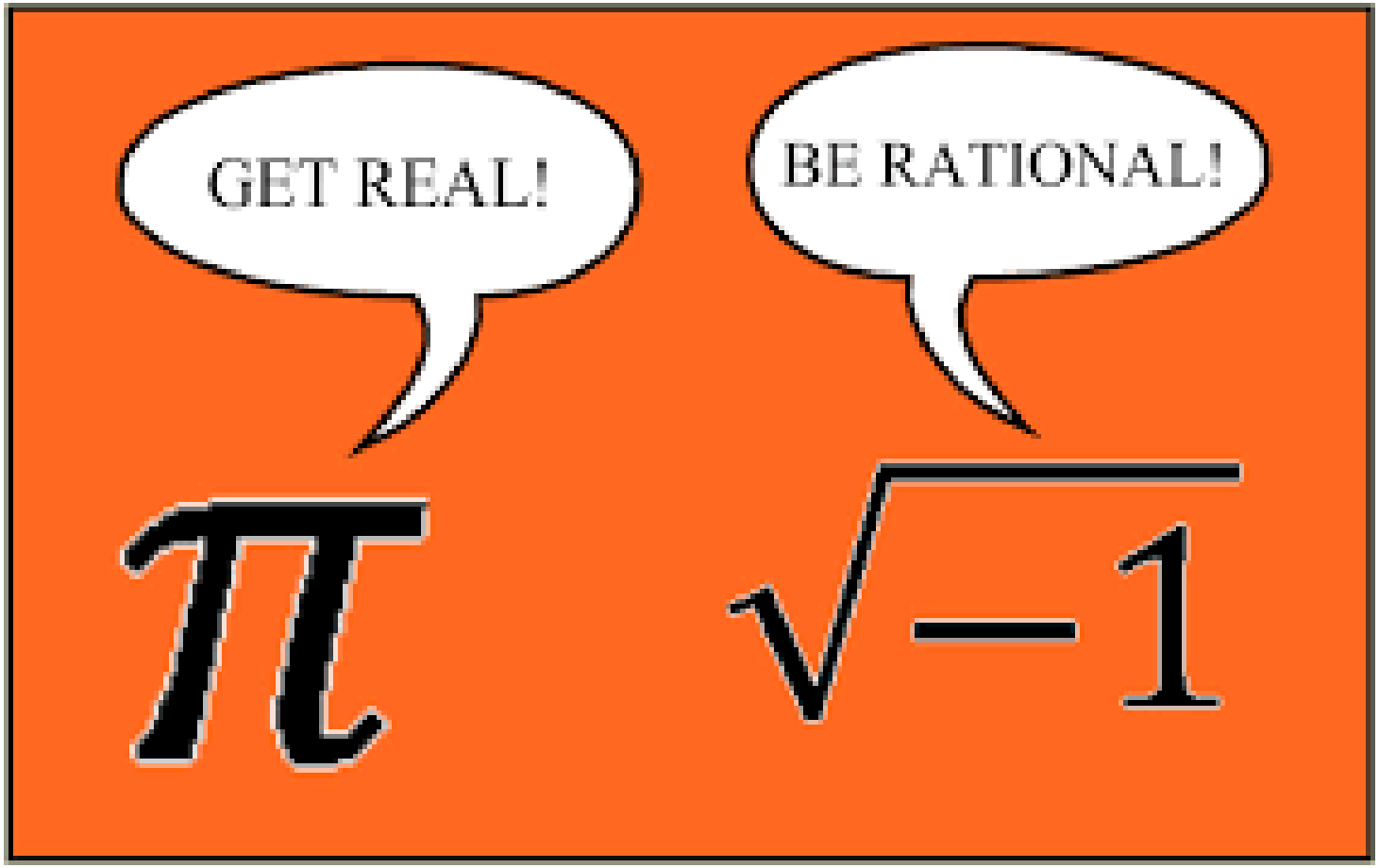
$\sqrt{-18} = 3\sqrt{2}i$

Radicals can't have a negative number (only $\sqrt{+x}$)

Infinity:

Infinity is the idea of something that has no end.

∞ $-\infty$


GET REAL!
BE RATIONAL!
π
√−1

What is a surd?

Vote on whether you think the following might be ‘surds’ or not surds.

$\sqrt{2}$	Not a surd	Surd
$\sqrt{9}$	Not a surd	Surd
$\sqrt{5}$	Not a surd	Surd
$\sqrt{\frac{1}{4}}$	Not a surd	Surd
$\sqrt[3]{7}$	Not a surd	Surd

Therefore, can you think of a suitable definition for a surd?

?

Laws of Surds

The only two things you need to know this topic…

$$\sqrt{a} \times \sqrt{b} = \boxed{?}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \boxed{?}$$

Basic Examples:

$\sqrt{3} \times \sqrt{2} =$?

$\sqrt{\frac{1}{9}} =$?

$\sqrt{4x^2} =$?

Simplifying Surds

$$\sqrt{8} = \boxed{?} = \boxed{?}$$

Could we somehow use $\sqrt{ab} = \sqrt{a}\sqrt{b}$ to break the 8 up in a way that one of the surds will simplify?

Fro Tip: Find the largest square factor of the number, and put that first.

$$\sqrt{27} = \boxed{?}$$

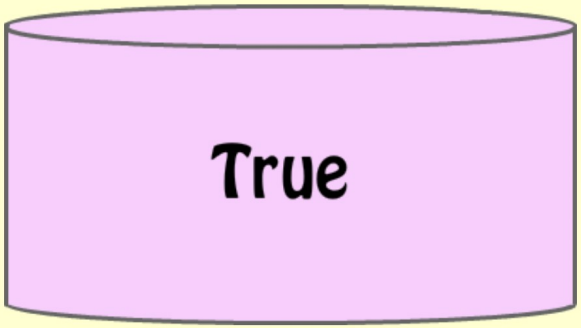
$$\sqrt{32} = \boxed{?}$$

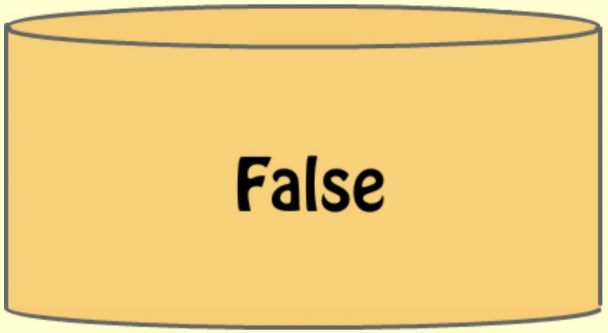
$$\sqrt{50} = \boxed{?}$$

$$\sqrt{12} = \boxed{?}$$

True or false?

$\sqrt{12} = 2\sqrt{3}$





Test Your Understanding So Far

a $\sqrt{24} =$

b $\sqrt{75} =$

c $\sqrt{20} =$

d $\sqrt{48} =$

Practise this specific Key Skill:
https://www.drfrostmaths.com/keyskills.php?permid=118

Independent practice

Fully simplify:

a $\sqrt{80}$

b $\sqrt{125}$

c $\sqrt{27}$

d $\sqrt{8}$

e $\sqrt{96}$

f $\sqrt{150}$

g $\sqrt{20}$

h $\sqrt{18}$

i $\sqrt{32}$

j $\sqrt{75}$

In each box, find pairs of numbers that are **equal** to each other.
Circle the number that is left over.

A

$\sqrt{12}$	$2\sqrt{5}$	$3\sqrt{2}$
$2\sqrt{2}$	$2\sqrt{3}$	$5\sqrt{2}$
$\sqrt{20}$	$\sqrt{18}$	$\sqrt{8}$

B

$4\sqrt{5}$	$4\sqrt{3}$	$\sqrt{27}$
$3\sqrt{3}$	$3\sqrt{5}$	$\sqrt{48}$
$\sqrt{45}$	$\sqrt{63}$	$\sqrt{80}$

C

$\sqrt{28}$	$2\sqrt{5}$	$3\sqrt{3}$
$\sqrt{32}$	$4\sqrt{2}$	$\sqrt{20}$
$2\sqrt{7}$	$\sqrt{27}$	$\sqrt{24}$

D

$\sqrt{40}$	$3\sqrt{10}$	$\sqrt{150}$
$5\sqrt{6}$	$\sqrt{50}$	$2\sqrt{10}$
$\sqrt{80}$	$4\sqrt{5}$	$\sqrt{90}$

E

$6\sqrt{2}$	$3\sqrt{11}$	$3\sqrt{5}$
$5\sqrt{3}$	$\sqrt{72}$	$\sqrt{75}$
$7\sqrt{2}$	$\sqrt{98}$	$\sqrt{99}$

F

$\sqrt{63}$	$4\sqrt{3}$	$2\sqrt{6}$
$\sqrt{48}$	$\sqrt{44}$	$3\sqrt{7}$
$\sqrt{18}$	$\sqrt{24}$	$2\sqrt{11}$

Multiples of Surds

$\sqrt{4} = 2$ and $6 \times 2 = 12$

$6\sqrt{20} =$?

$7\sqrt{12} =$?

$2\sqrt{45} =$?

Quick check in your tables

a $2\sqrt{75} =$?

b $3\sqrt{40} =$?

c $4\sqrt{48} =$?

d $3\sqrt{200} =$?

e $5\sqrt{45} =$?

Practise this specific Key Skill:
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Independent practice

Fully simplify:

a $4\sqrt{32}$

b $2\sqrt{45}$

c $2\sqrt{27}$

d $4\sqrt{50}$

e $4\sqrt{48}$

f $4\sqrt{27}$

g $2\sqrt{18}$

h $5\sqrt{24}$

i $5\sqrt{150}$

j $4\sqrt{45}$

4

$\sqrt{24}$

$\sqrt{8}$

$\sqrt{2}$

$3\sqrt{6}$

$\sqrt{5}$

$\sqrt{8}$

$\sqrt{10}$

$3\sqrt{6}$

$\sqrt{20}$

$\sqrt{2}$

$\sqrt{18}$	x	$\sqrt{3}$	=	
	x		=	$4\sqrt{3}$
	x		=	10
	x		=	$4\sqrt{5}$
	x	$\sqrt{2}$	=	
$\sqrt{27}$	x		=	

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multiplication squares.

x	$\sqrt{10}$	$\sqrt{6}$
$\sqrt{8}$	$\sqrt{}$	$2\sqrt{}$
$\sqrt{6}$	$2\sqrt{}$	

x	$\sqrt{9}$	$2\sqrt{3}$
$2\sqrt{3}$	$\_\_\sqrt{}$	
$3\sqrt{3}$	$\_\_\sqrt{}$	

x	$\sqrt{6}$	$\sqrt{3}$
$\sqrt{12}$	$\_\_\sqrt{8}$	
$\sqrt{7}$	$\sqrt{3} \times \sqrt{}$	$\sqrt{}$

x	$\sqrt{}$	$\sqrt{}$
$\sqrt{6}$	$\sqrt{48}$	$\sqrt{30}$
	$\sqrt{72}$	$\sqrt{45}$

Test Your Understanding

a $6 \times \sqrt{7} =$?

b $\sqrt{5} \times \sqrt{6} =$?

c $\sqrt{2} \times 2 =$?

d $5\sqrt{3} \times 4\sqrt{3} =$?

e $3 \times 4\sqrt{3} =$?

f $\left(3\sqrt{5}\right)^2 =$?

g $5\sqrt{8} \times 2\sqrt{2} =$?

h $2\sqrt{2} \times 3\sqrt{6} =$?

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https://www.drfrostmaths.com/keyskills.php?permid=122

Exercise 1

1 Simplify the following:

a $\sqrt{8} =$?

b $\sqrt{18} =$?

c $\sqrt{50} =$?

d $\sqrt{80} =$?

e $\sqrt{72} =$?

2 Simplify the following:

a $5\sqrt{80} =$?

b $2\sqrt{125} =$?

c $8\sqrt{12} =$?

d $3\sqrt{72} =$?

e $2\sqrt{28} =$?

3 Simplify the following:

a $\sqrt{3} \times \sqrt{2} \times \sqrt{5} =$?

b $\sqrt{27} \times \sqrt{3} =$?

c $4\sqrt{3} \times 2 =$?

d $5 \times 2\sqrt{5} =$?

e $2\sqrt{2} \times 2\sqrt{2} =$?

f $7\sqrt{3} \times 2\sqrt{5} =$?

g $6\sqrt{3} \times 2\sqrt{3} =$?

4 Simplify the following:

a $\sqrt{8} \times 3\sqrt{2} =$?

b $\sqrt{27} \times 2\sqrt{3} =$?

c $3\sqrt{18} \times \sqrt{2} =$?

d $2\sqrt{12} \times 3\sqrt{3} =$?

5 Express the following as a single square root

(hint: do the steps of simplification backwards!)

a $3\sqrt{2} =$?

b $2\sqrt{5} =$?

c $5\sqrt{7} =$?

d $4\sqrt{3} =$?

6 Express the following as a single square root:

a $a\sqrt{b} =$?

b $2\sqrt{k} =$?

Adding Surds

$$\sqrt{3} + \sqrt{3} = \boxed{?}$$

Think of it as "if I have one lot of $\sqrt{3}$ and I add another lot of $\sqrt{3}$, I have two lots of $\sqrt{3}$".
It's just how we collect like terms in algebra, e.g. $x + x = 2x$

$$2\sqrt{5} + \sqrt{5} = \boxed{?}$$

$$7\sqrt{7} + 7\sqrt{7} = \boxed{?}$$

$$\sqrt{2} + \sqrt{8} = \boxed{?}$$

$$= \boxed{?}$$

$$3\sqrt{12} + \sqrt{27} = \boxed{?}$$

$$= \boxed{?}$$

$$= \boxed{?}$$

Quick check in your tables

a $\sqrt{3}+\sqrt{3}+\sqrt{3} = ?$

b $\sqrt{8}+\sqrt{18} = ?$

c $2\sqrt{5}+2\sqrt{20} = ?$

d $3\sqrt{48}+\sqrt{12} = ?$

Practise this specific Key Skill:
https://www.drfrostmaths.com/keyskills.php?permid=119
https://www.drfrostmaths.com/keyskills.php?permid=120

$\sqrt{7}$

$\sqrt{5}$

$3\sqrt{7}$

$\sqrt{3}$

$\sqrt{12}$

$4\sqrt{2}$

$2\sqrt{6}$

$\sqrt{12}$

$\sqrt{54}$

$\sqrt{27}$

$2\sqrt{2}$	+	$\sqrt{8}$	=	
	+		=	$5\sqrt{3}$
$\sqrt{20}$	+		=	$3\sqrt{5}$
	+		=	$5\sqrt{6}$
	+		=	$\sqrt{27}$
	+	$\sqrt{28}$	=	

Complete these addition squares. Simplify your answers.

+	$3\sqrt{5}$	$-2\sqrt{5}$
$\sqrt{20}$		
$\sqrt{45}$		

+	$\sqrt{54}$	$3\sqrt{6}$
$\sqrt{24}$		
$-\sqrt{6}$		

$\sqrt{6}$ $2\sqrt{5}$

$\sqrt{20}$ 2

$\sqrt{18}$

$\sqrt{12}$ $\sqrt{5}$

$\sqrt{5}$ $2\sqrt{2}$

$2\sqrt{2}$

$\sqrt{18}$	-	$\sqrt{2}$	=	
$\sqrt{12}$	x		=	$4\sqrt{6}$
	+		=	$3\sqrt{5}$
	÷		=	2
$\sqrt{24}$	÷		=	
	x		=	$6\sqrt{6}$

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True or false?

$\sqrt{9} + \sqrt{4} = \sqrt{13}$

$\sqrt{3} \times \sqrt{3} = 3$

$\sqrt{9} \times \sqrt{4} = \sqrt{36}$

$\sqrt{2} \times \sqrt{2} \times \sqrt{2} = 2\sqrt{2}$

$\sqrt{200} = 10\sqrt{2}$

$\sqrt{200} = 2\sqrt{100}$

$\sqrt{200} = 2\sqrt{50}$

$\sqrt{2} = 1.4142136$

$\sqrt{2} = 1.414213562$

$\sqrt{2} = 1.4142$

$\sqrt{9} - \sqrt{4} = \sqrt{5}$

$\frac{\sqrt{36}}{\sqrt{4}} = \sqrt{9}$

$\sqrt{200} = 5\sqrt{8}$

$\sqrt{200} = \sqrt{20} \times \sqrt{10}$